Chapter 7. Cube and Cube Root

Question 1

Find the cubes of the following numbers: (i) 7, (ii) 12, (iii) 21, (iv) 100, (v) 302

Solution:

(i) $(7)^3 = 7\times7\times7 = 343$

(ii) $(12)^3 = 12 \times 12 \times 12 = 1728$

(iii) $(21)^3 = 21 \times 21 \times 21 = 9621$

(iv) $(100)^3 = 100 \times 100 \times 100 = 1000000$

(v) $(302)^3 = 302 \times 302 \times 302 = 27543608$

Question 2

Write cubes of all natural numbers between 1 and 20 and verify the following statements:

(a) Cubes of all odd natural numbers are odd.
(b) Cubes of all even natural numbers are even.

Solution:

$(2)^3 = 8$, $(3)^3 = 27$, $(4)^3 = 64$, $(5)^3 = 125$, $(6)^3 = 216$, ... $(19)^3 = 6859$.

(a) Yes, cubes of all odd natural numbers are odd.
(b) Yes, cubes of all even natural numbers are even.

Question 3

Write cubes of 5 natural numbers which are multiples of 3 and verify the following:

'The cube of natural number, which is a multiple of 3 is a multiple of 27'.

Solution:

$(3)^3 = 3\times 3\times 3 = 27$
$(6)^3 = 6\times 6\times 6 = 216$
$(9)^3 = 9\times 9\times 9 = 729$
$(12)^3 = 12\times 12\times 12 = 1728$
$(15)^3 = 15\times 15\times 15 = 3375$

Verification:

$(3)^3 = 27 = 27\times 1$
$(6)^3 = 216 = 27\times 8$
$(9)^3 = 729 = 27\times 27$
$(12)^3 = 1728 = 27\times 64$
$(15)^3 = 3375 = 27\times 125$

$\therefore$ 'The cube of natural number, which is a multiple of 3 is a multiple of 27'.

Question 4

Write cubes of 5 natural numbers which are of the form 3n+1 (e.g 4, 7, 10, ...) and verify the following: 'The cube of a natural number of the form 3n +1 is a natural number of the same form'.

Solution:

The 5 natural numbers which are of the form 3n + 1 (e.g 4, 7, 10, ...) are as follows:

$3 \times 1 + 1 = 3 + 1 = 4$

$3 \times 2 + 1 = 6 + 1 = 7$

$3 \times 3 + 1 = 9 + 1 = 10$

$3 \times 4 + 1 = 12 + 1 = 13$

$3 \times 5 + 1 = 15 + 1 = 16$

The cubes of 5 natural numbers which are of the form 3n + 1 (e.g 4, 7, 10, ...) are as follows:

$(4)^3 = 4 \times 4 \times 4 = 64$

$(7)^3 = 7 \times 7 \times 7 = 343$

$(10)^3 = 10 \times 10 \times 10 = 1000$

$(13)^3 = 13 \times 13 \times 13 = 2197$

$(16)^3 = 16 \times 16 \times 16 = 4096$

Verification:

$64 = 3 \times 21 + 1$

$343 = 3 \times 114 + 1$

$1000 = 3 \times 333 + 1$

$2197 = 3 \times 732 + 1$

$4096 = 3 \times 1365 + 1$

Question 5

Write cubes of 5 natural numbers which are of the form 3n + 2 (e.g. 5, 8, 11, ...) and verify the following: 'The cube of a natural number of the form 3n + 2 is a natural number of the same form'.

Solution:

The 5 natural numbers which are of the form 3n + 2 (e.g 5, 8, 11 , ...) are as follows:

$3 \times 1 + 2 = 3 + 2 = 5$

$3 \times 2 + 2 = 6 + 2 = 8$

$3 \times 3 + 2 = 9 + 2 = 11$

$3 \times 4 + 2 = 12 + 2 = 14$

$3 \times 5 + 2 = 15 + 2 = 17$

The cubes of 5 natural numbers which are of the form 3n + 2 (e.g 5, 8, 11, ...) are as follows:

$(5)^3 = 5 \times 5 \times 5 = 125$

$(8)^3 = 8 \times 8 \times 8 = 512$

$(11)^3 = 11 \times 11 \times 11 = 1331$

$(14)^3 = 14 \times 14 \times 14 = 2744$

$(17)^3 = 17 \times 17 \times 17 = 4913$

Verification:

$125 = 3 \times 41 + 2$

$512 = 3 \times 170 + 2$

$1331 = 3 \times 443 + 2$

$2744 = 3 \times 914 + 2$

$4913 = 3 \times 1637 + 2$

$\therefore$ 'The cube of a natural number of the form 3n + 2 is a natural number of the same form'.

Question 6

Which of the following numbers are perfect cubes? 1728, 106480

Solution:

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= (2^3 \times 2^3 \times 3^3) = (2 \times 2 \times 3)^3$

Therefore cube root of 1728 = $\sqrt[3]{1728} = 12$

Hence 1728 is a perfect cube.

2	106480
2	53240
2	26620
2	13310
5	6655
11	1331
11	121
	11

$106480 = 2 \times 2 \times 2 \times 2 \times 5 \times 11 \times 11 \times 11$

$= (2^3 \times 2 \times 5 \times 11^3) = (2 \times 11)^3 \times 2 \times 5$

In the above factorisation 2×5 remains after grouping in triplets.Therefore 106480 is not a perfect cube.

Question 7

What is the smallest number by which 392 must be multiplied so that the product is a perfect cube ?

Solution:

392 = 2 × 2 × 2 × 7 × 7

7 occurs as a prime factor only twice.

Hence, 7 is the smallest number by which 392 must be multiplied so that the product is a perfect cube.

2	392
2	196
2	98
7	49
	7

Question 8

What is the smallest number by which 8640 must be divided so that the quotient is a perfect cube ?

Solution:

8640 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5

5 occurs as a prime number only once.

Hence, 5 is the smallest number by which 8640 must be divided so that the quotient is a perfect cube.

2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
	5

Question 9

If one side of a cube is 13 metres, find its volume.

Solution:

The volume of a cube = $(\text{side})^3 = (13)^3 = 2197 m^3$.

Question 10

Find the cube root of:

(i) 343 (ii) 1000 (iii)2744 (iv) 74088

Solution:

(i) $343 = 7 \times 7 \times 7$

7	343
7	49
	7

$\therefore \sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$

(ii) $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

2	1000
2	500
2	250
5	125
5	25
	5

$\therefore \sqrt[3]{1000} = \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5} = 2 \times 5 = 10$

(iii) $2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$

2	2744
2	1372
2	686
7	343
7	49
	7

$\therefore \sqrt[3]{2744} = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7} = 2 \times 7 = 14$

(iv) $74088 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
	7

$\therefore \sqrt[3]{74088} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7} = 2 \times 3 \times 7 = 42$

Question 11

Find the cube root of 125.

Solution:

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 125
-  1
 124
-  7
 117
- 19
  98
- 37
  61
- 61
   0
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Since we had to subtract five times, therefore, $\sqrt[3]{125} = 5$

Question 12

Multiply 137592 by the smallest number so that the product is a perfect cube. Also, find the cube root of the product.

Solution:

137592 = 2′2′2′3′3′3′7′7′13

The number 7 and 13 should be multiplied once and twice respectively so that the product is a perfect cube.

∴ The smallest number by which 137592 must be multiplied = 7′13′13 = 1183

The required product = 137592 x 1183 = 2 x 2 x 2 x 3 x 3 x 3 x 7 x 7 x 13 x 7 x 13 x 13

$= (2^3 \times 3^3 \times 7^3 \times 13^3)$

$= (2 \times 3 \times 7 \times 13)^3$

$\sqrt[3]{137592 \times 1183} = 2 \times 3 \times 7 \times 13$

$= 546$

Question 13

Divide the number 26244 by the smallest number so that the quotient is a perfect cube. Also, find the cube root of the quotient.

Solution:

2	26244
2	13122
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
	3

$26244 = 2\times2\times3\times3\times3\times3\times3\times3\times3\times3$

$2\times2\times3\times3 = 36$ is the smallest number by which 26244 must be divided so that the quotient is a perfect cube.

$$
\begin{array}{r|l}
 & 729 \\
36 & 26244 \\
 & \underline{252} \\
 & 0104 \\
 & \underline{0072}
\end{array}
$$

Question 14

The volume of a cube is 512cubic metres. Find the length of the side of the cube.

Solution:

We know that, the volume of a cube = $(\text{side})^3$

The length of the side of a cube = $\sqrt[3]{512} = \sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2} = 2\times2\times2 = 8\,\text{m}$

Question 15

Which of the following numbers are cubes of negative integers?

(a) -64 (b) -2197 (c) -1056 (d) -3888

Solution:

(a) 64 = 2x2x2x2x2x2

$$\sqrt[3]{64} = \sqrt[3]{2\times 2\times 2\times 2\times 2\times 2} = 2\times 2 = 4$$

$$\sqrt[3]{-64} = -\sqrt[3]{64} = -4$$

$\therefore$ -64 is a cube of -4 a negative integer.

(b) 2197 = 13x13x13

$$\sqrt[3]{2197} = \sqrt[3]{13\times 13\times 13} = 13$$

$$\sqrt[3]{-2197} = -\sqrt[3]{2197} = -13$$

$\therefore$ -2197 is a cube of -13 a negative integer.

(c) 1056 = 2x2x2x2x2x3x11

In the above factorisation 2 x 3 x 3 x 11 x 11 remains after grouping in triplets. Therefore, 1056 is not a perfect cube.

Hence -1056 is not a cube of negative integer.

(d) 3888 = 2 x 2 x 2 x 2 x 3 x 3 x 3 x 3 x 3

In the above factorisation 2 x 3 x 3 remains after grouping in triplets. Therefore, 3888 is not a perfect cube.

$$\sqrt[3]{5832} = \sqrt[3]{2\times 2\times 2\times 3\times 3\times 3\times 3\times 3\times 3} = 2\times 3\times 3 = 18$$

$$\sqrt[3]{-5832} = -\sqrt[3]{5832} = -18$$

Question 16

Find the cube roots of:

(a) -125 (b) -5832 (c) -17576

Solution:

(a) 125 = 5x5x5

$$\sqrt[3]{125} = \sqrt[3]{5\times 5\times 5} = 5$$

$$\sqrt[3]{-125} = -\sqrt[3]{125} = -5$$

(b) 5832 = 2x2x2x3x3x3x3x3x3

$$\sqrt[3]{5832} = \sqrt[3]{2\times 2\times 2\times 3\times 3\times 3\times 3\times 3\times 3} = 2\times 3\times 3 = 18$$

$$\sqrt[3]{-5832} = -\sqrt[3]{5832} = -18$$

(c) 17576 = 2x2x2x13x13x13

$$\sqrt[3]{17576} = \sqrt[3]{2\times 2\times 2\times 13\times 13\times 13} = 2\times 13 = 26$$

$$\sqrt[3]{-17576} = -\sqrt[3]{17576} = -26$$

Question 17

Find the cube root of each of the following numbers:

1. 8 ×64
2. (-216) × 1728
3. 27 × (-2744)
4. (-125)×(-3375)
5. -456533
6. -5832000

Solution:

(1) 8 x 64 = 2x2x2x2x2x2x2x2x2

$\sqrt[3]{8\times 64} = 2\times 2\times 2 = 8$

(2) 216 x 1728 = 2x2x2x3x3x3x2x2x2x2x2x2x3x3x3

$\sqrt[3]{(-216)\times 1728} = -(2\times 3\times 2\times 2\times 3) = -72$

(3) 27 x 2744 = 3x3x3x2x2x2x7x7x7

$\sqrt[3]{27\times(-2744)} = -(3\times 2\times 7) = -42$

(4) 125 x 3375 = 5x5x5x3x3x3x5x5x5

$\sqrt[3]{(-125)\times(-3375)} = 5\times 3\times 5 = 75$

(6) 5832000 = 5832 x 1000 = 2x2x2x3x3x3x3x3x3x2x2x2x5x5x5

$\sqrt[3]{(-5832)\times 1000} = -(2\times 3\times 3\times 2\times 5) = -180$

Question 18

Find the cubes of the following by multiplication.

(i) -4
(ii) 23
(iii) 3030

Solution:

(i) $(-4)^3$ =(4) x (4) x (4) = -64

(ii) $(23)^3$ =23 x 23 x 23 = 12167

(iii) $(3030)^3$ =3030 x 3030 x 3030 = 27818127000.

Question 19

Find the cube of the following rational numbers:

(i) 1.4

Solution:

(i) $(1.4)^3 = 1.4 \times 1.4 \times 1.4 = 2.744$.

Question 20

By what number would you multiply 231525 to make it a perfect cube?

Solution:

The prime factorisation of 231525 is $5 \times 5 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$.

The number that must be multiplied in order that the above product is a perfect cube is 5.

Therefore, Cube root of 231525×5 is $5 \times 3 \times 7 = 105$.